Int. J. Multiphase Flow, Vol. 2, pp. 171–182. Pergamon/Elsevier, 1975. Printed in Great Britain

HEAT AND MASS TRANSFER IN THE VICINITY OF AN EVAPORATING DROPLET

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(*Received* 1 *September* 1974)

Abstract—The problem of heat and mass transfer around an evaporating water drop located in a superheated steam flow is studied using singular perturbation methods.

The perturbation scheme involves three regions and some results concerning the Nusselt number and the drag coefficient of the drop are presented.

INTRODUCTION

The study of heat and mass transfer in the vicinity of an evaporating liquid droplet moving in the vapor of the same fluid is of great interest for a fundamental approach of some phenomena encountered in power plants. For instance, in certain types of desuperheaters, water drops are injected into the core of a superheated steam flow in order to control the temperature and a knowledge of the behaviour of drops injected in such conditions is important.

It is particularly interesting to compute the drag coefficient and the Nusselt number of an evaporating droplet in order to evaluate the momentum and energy interaction terms between both phases of a two-phase flow.

A great many studies concerning drops have already been performed but most of them deal with drops moving in an atmosphere different from the vapor corresponding to the liquid. In such cases the phenomena involved are essentially diffusion phenomena. Very few authors (Ross 1966; Fendell *et al.* 1966) have tried to analyse the pure multiphase thermal transfer problem which is going to be considered; generally, they assumed the form of the velocity field around the drop and solved the heat transfer problem. In fact, the dynamic problem and heat transfer problem are strongly connected through the interface conditions at the surface of the drop and it is certainly incorrect to study one problem independently of the other. A correct analysis of the interactions between a drop and the surrounding vapor implies the determination of the velocity and temperature fields near the surface of the drop. So, an attempt is made here to set up a method which leads to a complete solution of the governing equations of a simple model elaborated with suitable assumptions.

STATEMENT OF THE PROBLEM

Consider a drop of liquid in a uniform flow of the corresponding steam, the properties of which (temperature, pressure, velocity) are known far from the drop, and try to analyse the steam flow as well as the heat and mass transfer phenomena involved. More precisely, we will attempt to determine the velocity and temperature fields around this drop in order to compute the drag coefficient and the Nusselt number. These coefficients express the result of the dynamic and thermal interactions of this drop with the surrounding steam.

A reference system, connected with the center of the drop is used. The coordinates are axisymmetric; r is the vector radius of a point, ϕ is its colatitude and σ denotes $\cos\phi$. In this reference system, the vapor velocity at infinity is uniform and the problem is axisymmetric; the velocity and temperature fields may be determined in a meridian plane.

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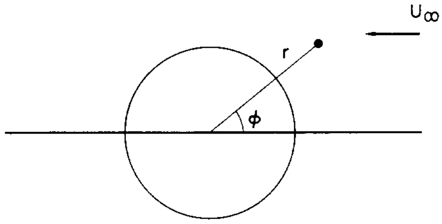

The following assumptions are made:

At the interface, thermodynamic equilibrium conditions prevail. The steam pressure is thus not very different from the pressure at infinity and the interface temperature equals the saturation temperature corresponding to this pressure (Montlucon 1972).

The drop remains spherical and the temperature of the liquid is uniform and equal to the saturation temperature.

This hypothesis is valid only if the following inequalities apply:

$$Re \triangleq \frac{\rho U_\infty 2R}{\eta} \gg 1; \quad We \triangleq \frac{\rho U_\infty^2 2R}{\sigma} \ll 1; \quad Fr \triangleq \frac{U_\infty^2}{g2R} \gg 1$$

which is the case for very small droplets ($2R \sim 50$–$100\ \mu$m). Physical properties of the vapor are considered to be constant. There is no buoyancy and, in our frame of reference, the liquid velocity is negligible compared with the steam velocity. Buoyancy effects can be neglected if the following inequality applies:

$$Fr' \triangleq \frac{U_\infty^2}{\dfrac{T_\infty - T_s}{T_\infty} g2R} \gg 1$$

which is the case for very small droplets ($2R \sim 50$–$100\ \mu$m). The evolution of the system may be represented by a succession of steady states. The variations in the drop radius R and in the relative velocity may be determined from velocity and temperature fields which are solutions of the stationary problem whose parameters are the instantaneous parameters of the real system. Consequently, partial derivatives with respect to time may be neglected in the governing equations and stationary boundary conditions may be considered. In a previous study, Montlucon (1972) showed that, for a motionless droplet evaporating in superheated steam, the analytical solution based on a succession of steady states was in good agreement with the numerical solution of the complete equations. This assumption is now extended to a moving droplet.

These hypotheses are valid for water drops evaporating in superheated steam. A complete justification may be found in Montlucon (1972).

METHOD OF SOLUTION

On the basis of the above assumptions, we are dealing with the determination of the velocity and temperature fields in the steam surrounding the drop. For this axisymmetric problem, the Navier–Stokes equations and the energy equation may be written in a classical form which is used by Berker (1963) and Fendell *et al.* (1966):

$$\left.\begin{aligned} \nu D^4\psi + (1-\sigma^2)L(\psi,\psi) = 0 \\ \alpha \nabla^2 T + \frac{1}{r^2}H(\psi, T) = 0 \end{aligned}\right\}. \qquad [1]$$

ψ is the Stokes stream function and T the temperature of the steam r and σ are the coordinates of a point in a meridian plan as represented in reference system ($\sigma = \cos\varphi$). ν represents the kinematic viscosity of the steam and α its thermal diffusivity. D^2, L, ∇^2 and H are functionnal operators defined below:

$$D^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{1-\sigma^2}{r^2}\frac{\partial^2 f}{\partial \sigma^2} \quad H(f, g) = \frac{D(f, g)}{D(r, \sigma)} \text{ (Jacobian)}$$

$$L(f, g) = H\left(f, \frac{D^2 g}{r^2(1-\sigma^2)}\right) \qquad \nabla^2 f = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{\partial}{\partial \sigma}\left((1-\sigma^2)\frac{\partial f}{\partial \sigma}\right)\right].$$

The system [1] is completed by the following boundary conditions:

$$T(R, \sigma) = T_s \quad \frac{\partial \psi}{\partial r}(R, \sigma) = 0 \quad \lambda \frac{\partial T}{\partial r}(R, \sigma) = \rho \frac{\Delta H}{R^2}\frac{\partial \psi}{\partial \sigma}(R, \sigma)$$

$$\lim_{r\to\infty} T(r, \sigma) = T_\infty \qquad \lim_{r\to\infty} \psi(r, \sigma) = U_\infty r^2 \frac{(\sigma^2 - 1)}{2}.$$

R is the radius of the drop. λ and ρ are the conductivity and the density of the steam, respectively. ΔH represents the latent heat of vaporisation and U the steam velocity. Subscript s characterizes the surface of the drop and subscript ∞ characterizes infinity.

In order to find an analytical solution for this system, a singular perturbation method may be used. Such a procedure is now classical and our notations are those adopted by Van Dyke (1964). A solution is sought in the form of a set of asymptotic expansions in relation to a small parameter involving the steam velocity far from the drop. Every expansion is valid in a region D_i which may be scaled by the order of magnitude $L_{0,i}$ of the vector radius of one of its points. A physical interpretation of the regions D_i is given in the next section. In a region D_i, the unknown variables of the problem ψ and $T - T_s$ have the order of magnitude $\psi_{0,i}$ and $T_{0,i}$. From a general point of view, a region D_i and the solution which is valid in it, may be characterized by three dimensionless parameters:

$$\theta_i = \frac{L_{0,i}}{R} \quad \xi_i = \frac{\alpha L_{0,i}}{\psi_{0,i}} \quad \tau_i = \frac{T_{0,i}}{T_\infty - T_s}.$$

In a region D_i, [1] may be written in a dimensionless form:

$$\left.\begin{aligned} \xi_i D_i^4 \psi_i + Pr^{-1}(1-\sigma^2) L_i(\psi_i, \psi_i) = 0 \\ \xi_i \nabla_i^2 T_i + \frac{1}{r_i^2} H_i(\psi_i, T_i) = 0 \end{aligned}\right\} \qquad [2]$$

where

$$r_i = \frac{r}{L_{0,i}} \quad \psi_i = \frac{\psi}{\psi_{0,i}} \quad T_i = \frac{T - T_s}{T_{0,i}}. \qquad [3]$$

The functional operators D_i^2, H_i, L_i and ∇_i^2 are identical to the functional operators D^2, H, L and ∇^2 but the dimensionless variable r_i is used instead of r.

System [2] must be completed with boundary conditions which, according to the region D_i under consideration, may be matching conditions between asymptotic expansions or boundary conditions deduced from the boundary conditions of [1].

When writing of systems analogous to [2] and corresponding boundary conditions for the different regions D_i which must be considered in order to find a complete solution, three dimensionless quantities appear which are characteristics of the physical problem.

The Prandtl number $Pr = \nu/\alpha$ is a property of the fluid.

The superheat parameter $B = (H_\infty - H_s)/\Delta H$, where H represents the specific enthalpy of the steam, is characteristic to the thermal conditions of the problem. It appears in the boundary conditions on the surface of the drop.

In the case of evaporation of water drops in superheated steam which is the problem under consideration, the Prandtl number and the superheat parameter are fixed and of the order of unity. The third dimensionless quantity is the Peclet number $Pe = RU_\infty/\alpha$, which represents the dynamic conditions of the problem and is used as the small parameter of the different asymptotic expansions.

The first order approximations are solutions of the equations obtained by letting the small parameter Pe tend towards zero in [2]. It can be expected, as in most perturbation problems, that these equations have a simplified form compared to that of the governing equations of the problem. When Pr is fixed, [2] depend only on the dimensionless parameter ξ_i characteristic of the region of validity D_i. The only possible "simplified forms" of the governing equations are presented in table 1.

Table 1. Simplified forms of the governing equations

$\xi_i = o(1)$	$\xi_i = \text{Ord}\,(1)$	$\frac{1}{\xi_i} = o(1)$
Ideal fluid equation Convection equation	Navier–Stokes equation Conduction–convection equation	Stokes equation Conduction equation

Stokes equation (ideal fluid equation) obviously names the equation verified by the Stokes stream function when the inertia terms (the viscous terms) are neglected. Moreover, the conduction equation (convection equation) names the equation verified by the temperature when convection terms (conduction terms) are neglected.

Fendell *et al.* (1966) studied heat transfer around a sphere with mass transfer at the surface of the sphere. In their solution, the "simplified form" of the governing equations whose solution is a first order approximation in a region next to the sphere is made up of a Stokes equation and a conduction–convection equation. According to table 1, this is not a correct scheme.

For the problem which is examined here, the solution corresponding to a zero velocity far from the drop is classical:

$$\left.\begin{aligned} \psi &= \alpha R \log(1+B)(1+\sigma) \\ \frac{T-T_s}{T_\infty - T_s} &= \frac{1+B}{B}\left[e^{-\frac{R}{r}\log(1+B)} - e^{-\log(1+B)}\right] \end{aligned}\right\}. \tag{4}$$

Equations [4] written in dimensionless form are the first terms of asymptotic expansions which are valid in a region D_1 next to the drop. In this region, $L_{0,1} = R$, $\psi_{0,1} = \alpha R$ and $T_{0,1} = T_\infty - T_s$. D_1 and the asymptotic expansion valid in D_1 are characterised by the three dimensionless parameters

$$\xi_1 = 1, \quad \theta_1 = 1, \quad \tau_1 = 1.$$

Table 1 shows that the "simplified form" of the governing equations is actually not simplified at all. The first order approximation is a solution of the complete equations and only the spherical symmetry enables this external problem (Van Dyke 1964) to be completely determined.

This first approximation is not valid throughout the region around the drop because the boundary conditions relating to the Stokes stream function cannot be satisfied. The problem under consideration is in fact a singular perturbation problem; so another asymptotic expansion, valid in another region $D_2' \neq D_1$ characterized by three dimensionless parameters

$$\xi_2 = \frac{\alpha L_{0,2}}{\psi_{0,2}}, \quad \theta_2 = \frac{L_{0,2}}{R}, \quad \tau_2 = \frac{T_{0,2}}{T_\infty - T_s},$$

must be determined.

In this new region, the first order approximation of this new asymptotic expansion is a solution of equations having a "simplified form" and satisfies, on one hand, the boundary conditions at infinity and on the other hand, the matching conditions with the first order approximation of the previous asymptotic expansion valid in D_1. In order to determine the "simplified form" of the governing equations which must be considered, the consequences of the existence of an intermediate region D_m in which the two expansions are simultaneously valid are examined (Francois 1969). D_m and the solution which is valid in it are characterized by the three parameters ξ_m, θ_m and τ_m. Assume that:

$$\psi_1 = \mathrm{Ord}\,(F_1(\sigma) r_1^{m_1}) \quad \text{and} \quad T_1 = \mathrm{Ord}\,(G_1(\sigma) r_1^{n_1}) \quad \text{when} \quad r_1 \to \infty$$

$$\psi_2 = \mathrm{Ord}\,(F_2(\sigma) r_2^{m_2}) \quad \text{and} \quad T_2 = \mathrm{Ord}\,(G_2(\sigma) r_2^{m_2}) \quad \text{when} \quad r_2 \to 0.$$

The numbers m_1, n_1, the functions $F_1(\sigma)$ and $G_1(\sigma)$ are known and the corresponding numbers and functions m_2, n_2, $F_2(\sigma)$ and $G_2(\sigma)$ are sought. The different asymptotic expansions are expressed in their common region of validity D_m using the corresponding dimensionless space variable r_m.

$$\frac{L_{0,1}}{\xi_1} F_1(\sigma) \left(\frac{\theta_m}{\theta_1}\right)^{m_1} r_m^{m_1} = \frac{L_{0,2}}{\xi_2} F_2(\sigma) \left(\frac{\theta_m}{\theta_2}\right)^{m_2} r_m^{m_2}$$

$$T_{0,1} G_1(\sigma) \left(\frac{\theta_m}{\theta_1}\right)^{n_1} r_m^{n_1} = T_{0,2} G_2(\sigma) \left(\frac{\theta_m}{\theta_2}\right)^{n_2} r_m^{n_2}.$$

Hence, the following relations may be inferred:

$$\left.\begin{array}{cc} m_1 = m_2 & n_1 = n_2 \\ F_1(\sigma) = F_2(\sigma) & G_1(\sigma) = G_2(\sigma) \\ \dfrac{\xi_1}{\xi_2}\left(\dfrac{\theta_1}{\theta_2}\right)^{m_1 - 1} = 1. & \dfrac{\tau_2}{\tau_1}\left(\dfrac{\theta_1}{\theta_2}\right)^{n_1} = 1\ . \end{array}\right\} \qquad [5]$$

In the present case, $m_1 = n_1 = 0$, $\xi = \theta_1 = \tau_1 = 1$ and, in the domain D_2', $\psi_{0,2} = U_\infty L_{0,2}^2$. The parameters characteristic of the region D_2', and of the solution which is valid in it may now be inferred:

$$\xi_2 = Pe^{-1/2} \quad \theta_2 = Pe^{-1/2} \quad \tau_2 = 1.$$

$1/\xi_2 = o(1)$ and table 1 shows that the suitable simplified form of the governing equations is necessarily made up of a Stokes equation and a pure conduction equation. The first order approximations of the two asymptotic expansions valid in the two regions D_1 and D_2' may be written:

in the region D_1:

$$\left.\begin{array}{l} \psi_1 = \log(1+B) Q_0(\sigma) + o(Pe) \\ T_1 = \dfrac{1+B}{B}\left[e^{-[(\log 1+B)/r_1]} - e^{-\log 1+B}\right] + o(Pe) \end{array}\right\} \qquad [6]$$

in the region D_2':

$$\left.\begin{array}{l}\psi_2 = \log(1+B)Q_0(\sigma) + (Cr_2 + r_2^2)Q_1(\sigma) + o(Pe)\\ T_2 = 1 + o(Pe)\end{array}\right\}, \qquad [7]$$

where C is an integration constant which is determined later and $Q_i(\sigma)$ are Gegenbauer polynomials.

Approximations of higher orders of these two asymptotic expansions may be constructed but the classical Whitehead paradox is then encountered and it is impossible to determine an approximation of order Pe satisfying the boundary conditions at infinity. The determination of an asymptotic expansion valid in the region D_2' must be replaced by the determination of two asymptotic expansions valid respectively in two regions D_2 and D_3 ($D_2' = D_2 \cup D_3$) and satisfying boundary conditions and suitable matching conditions.

Using the method which leads to [5] and replacing the subscripts 1 and 2 by subscripts 2 and 3, the parameters characterizing these new regions and the solutions which are valid in them may be inferred

$$\left.\begin{array}{l}\psi_3 = r_3^2 Q_1(\sigma) + o(Pe)\\ T_3 = 1 + o(Pe)\end{array}\right\}. \qquad [8]$$

The first order approximation of the complete solution valid around the drop is made up of the set of functions [6], [7], [8]. The "simplified forms" of the governing equations verified by these functions are a Stokes equation and a pure conduction equation in the intermediate region of validity D_2 and complete equations in the extreme regions of validity D_1 and D_3. The problem being considered is thus a problem of singular perturbation with three regions. A physical interpretation of this conclusion is given below.

PHYSICAL INTERPRETATION

Consider the governing energy equation:

$$\alpha \nabla^2 T + \frac{1}{r^2} H(\psi, T) = 0.$$

In a region D_i characterized by the orders of magnitude $L_{0,i}$, $\psi_{0,i}$, $T_{0,i}$, the ratio of the convection term to the conduction term has the following order of magnitude:

$$\frac{\text{Conduction term}}{\text{Convection term}} = \text{Ord}\left[\frac{L_{0,i}\alpha}{\psi_{0,i}}\right] = \xi_i.$$

In the region D_1, this ratio equals 1; the conduction term and the convection term retain the same order of magnitude when Pe tends towards zero. In the region D_2, this ratio equals $Pe^{-1/2}$; the conduction term is predominant when Pe tends towards zero.

In the region D_3, this ratio equals the Prandtl number. For Prandtl numbers which are of the order of unity as those examined here, the conduction term and the convection term retain the same order of magnitude when Pe tends towards zero.

A similar analysis may be performed for the dynamic equation and the ratio of the viscosity term versus the inertia term has the order of magnitude

$$\frac{\text{Viscosity term}}{\text{Interia term}} = \text{Ord}\left[\frac{L_{0,i}\nu}{\psi_{0,i}}\right] = \xi_i Pr^{-1}.$$

$$\xi_2 = Pe^{-1/2}, \quad \theta_2 = Pe^{-1/2}, \quad \tau_2 = 1$$

$$\xi_3 = Pr^{-1}, \quad \theta_3 = PrPe^{-1}, \quad \tau_3 = 1.$$

In the region D_2 the "simplified form" of the governing equations is the same as the simplified form determined in D_2'. The corresponding first order approximation is [7].

In the region D_3 the "simplified form" of the governing equations is not more simplified than in the domain D_1. In this case the solution of these equations is trivial and accounts for a uniform flow with uniform temperature.

In the problem considered here, $Pr = \mathrm{Ord}(1)$ and the following conclusions may be reached:

In the region D_2, the viscosity term is predominant and in the regions D_1 and D_3, the viscosity term and inertia term are of the same order of magnitude.

It is thus natural to divide the region around the drop into three regions: D_1, D_2, D_3 scaled respectively by the reference lengths R, $RPe^{-1/2}$ and $RPe^{-1}Pr$. In the region D_2 the viscous and conductive transfers are predominant and in the regions D_1 and D_3 they are of the same order of magnitude as inertia and convective transfers.

These conclusions are incorrect if $Pr \neq \mathrm{Ord}(1)$. In this case it is necessiary to consider a fourth region D_4 scaled by a new reference length RPe^{-1}.

THE SOLUTION

A solution to the problem is determined in the form of six asymptotic expansions with respect to the Peclet number of the drop. These six asymptotic expansions (three for the Stokes stream function and three for the temperature) satisfy the matching conditions and the boundary conditions of the initial problem. The gauge functions $\nu_j(Pe)$ are progressively determined with the solution.

In the three regions D_1, D_2, D_3, [2] are written replacing ψ_i and T_i by their asymptotic expansions:

$$\psi_i = \Sigma\, \psi_{i,j}\nu_j(Pe) \quad T_i = \Sigma\, T_{i,j}\nu_j(Pe).$$

Three sets of systems of partial differential equations with r and σ are thus obtained. A decomposition of the variables on a complete base of orthogonal functions of σ makes it possible for every successive approximation to deal with differential equations in the space variable r_i. The base of Legendre polynomials $P_k(\sigma)$, the eigenfunctions of the operator ∇^2, is used in the determination of the temperature. The base of Gegenbauer polynomials, the eigenfunctions of the operator D^2 is used for the determination of the Stokes stream function.

The resolution of these sets of equations is particularly tedious and was carried out up to the order Pe^2 (inclusive for the temperature, exclusive for the Stokes stream function). Details of the solution may be found in Montlucon (1972) and are not given here.

The first terms of the asymptotic expansions valid in the region D_1 next to the sphere have been determined

$$\left.\begin{aligned} T_1 &= \sum_k \sum_j T_{1,k,j}(r_1)\nu_j(Pe)P_k(\sigma) \\ \psi_1 &= \sum_k \sum_j \psi_{1,k,j}(r_1)\nu_j(Pe)Q_k(\sigma). \end{aligned}\right\} \qquad [9]$$

These expansions are essential in order to determine the total interaction quantities that are the Nusselt number of the drop and its drag coefficient.

As far as the temperature is concerned, only the component $T_{1,0}$ is necessary for calculating the Nusselt number of the drop:

$$\left.\begin{aligned} T_{1,0} = \frac{1+B}{B}\Big[& e^{-[\log(1+B)/r_1]}\Big(1 - e^{-\log(1+B)(1-1/r_1)} + \log(1+B) \\ & \cdot\left(1-\frac{1}{r_1}\right)\left(\frac{Pe}{2} - \frac{H_1(BPr)}{3}Pe^2 \log Pe\right) + T_{1,0,5}(B, Pr, r_1)Pe^2 + O(Pe^3 \log Pe)\Big)\Big]. \end{aligned}\right\} \qquad [10]$$

$$\left.\begin{aligned}\psi_1 = \log(1+B)Q_0(\sigma) + \left[\frac{\log(1+B)}{2}Q_0(\sigma) + \left(\frac{H_2(B,Pr)}{r_1} + r_1^2 + H_1(B,Pr)Z(r_1)\right)Q_1\sigma\right]Pe \\ - \frac{\log(1+B)}{3}H_1(B,Pr)Q_0(\sigma)Pe^2\log Pe + O(Pe^2)\end{aligned}\right\} \quad [11]$$

where:

$$Z(r) = r + a_1 + a_2\frac{\log r}{r} + \sum_{n=3}^{\infty}\frac{a_n}{r^n}$$

with:

$$a_1 = -\frac{3}{4}\frac{\log(1+B)}{Pr} \quad a_2 = -\frac{4}{15}a_1\frac{\log(1+B)}{P_r} \quad a_n = \frac{(n-3)(n+2)}{(n-2)(n+3)(n+1)}\frac{\log(1+B)}{P_r}a_{n-1}.$$

The functions $H_1(B, P_r)$, $H_2(B, Pr)$, $T_{1,0,5}(B, P_r, r_1)$ are numerically determined during the construction of the asymptotic expansions (Montlucon 1972).

RESULTS

Determination of the Nusselt number

The Nusselt number which is representative of the thermal interactions between the drop and the surrounding steam is calculated from:

$$Nu = \int_{-1}^{+1}\frac{\partial T_1}{\partial r_1}(1,\sigma)\,d\sigma = 2\sum_j \frac{dT_{1,0,j}(1)}{dr_1}\nu_j(Pe).$$

Using [10], the following is obtained:

$$Nu = 2\frac{\log 1+B}{B}\left[1 + \frac{Pe}{2} - \frac{H_1(B,Pr)}{3}Pe^2\log Pe + H_3(B,Pr)Pe^2\right] + O(Pe^3\log Pe).$$

The functions H_1 and H_3 were computed and their variations with respect to the superheat parameter B for different values of the Prandtl number Pr are presented on figures 1 and 2. The

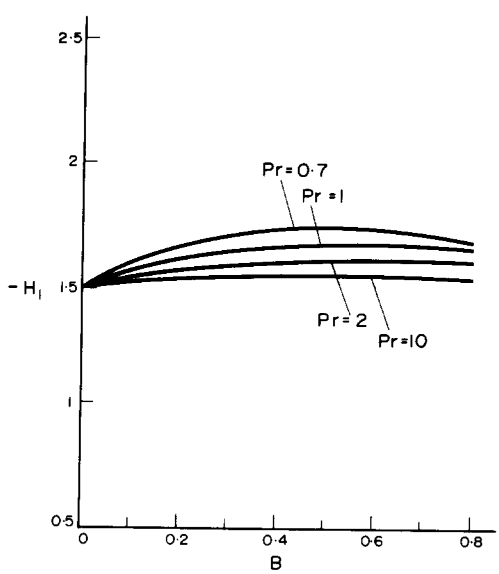

Figure 1. Values of the coefficient H_1 as a function of the superheat parameter B and the Prandtl number Pr.

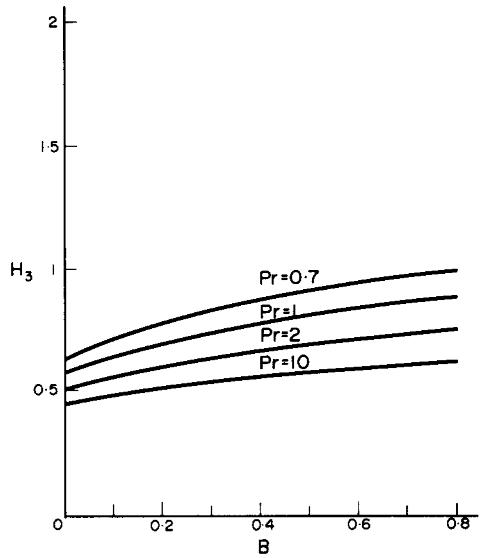

Figure 2. Values of the coefficient H_3 as a function of the superheat parameter B and the Prandtl number Pr.

conclusion of the analysis made by Gupalo & Ryasantsev (1972) are obtained in the case where $Pr = 1$ and $B = 0$ (no mass transfer).

$$H_1(0, 1) = -\frac{3}{2} \quad H_3(0, 1) = 0.592.$$

The total variation of the Nusselt number as a function of the superheat parameter B is presented in figure 3 for different values of the Peclet number Pe in the case $Pr = 1$.

Determination of the resistance coefficient

The resultant **F** of the forces acting on the drop in this axisymmetric problem is parallel to $\mathbf{U}_\infty$ and has the following form.

$$F = 2\pi\eta\alpha \int_{-1}^{+1} \left[\frac{\partial^2\psi_1}{\partial\sigma^2}(1, \sigma) - \frac{\partial^2\psi_1}{\partial r_1^2}(1, \sigma) - \left(Pr^{-1} \left(\frac{\partial\psi_1}{\partial\sigma}(1, \sigma) \right)^2 + P_1(1, \sigma) + 4 \frac{\partial\psi_1}{\partial\sigma}(1, \sigma) \right) \sigma - \frac{\partial^2\psi_1}{\partial\sigma^2}(1, \sigma)\sigma^2 \right] d\sigma$$

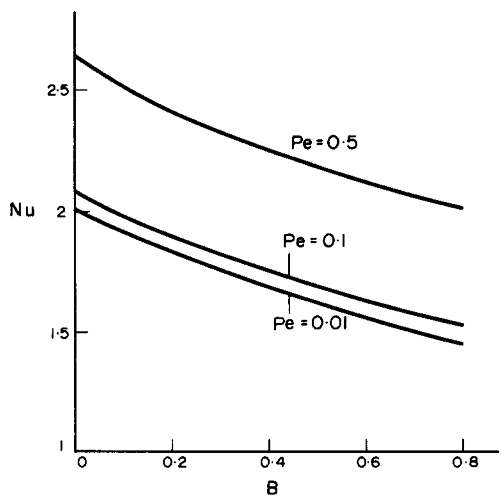

Figure 3. Nusselt number as a function of the superheat parameter B and the Peclet number $Pe(Pr = 1)$.

η represents the dynamic viscosity of the steam and $p_1 = pR^2/\eta\psi_{0,1}$ is a dimensionless form of the pressure p. This resultant force is divided into three parts $F = F_1 + F_2 + F_3$. F_1 is the result of the pressure forces acting on the drop:

$$F_1 = 2r\eta\alpha \int_{-1}^{+1} -P_1(1, \sigma)\sigma \, d\sigma.$$

F_2 is the result of the mass transfer forces acting on the drop.

$$F_2 = 2\pi\eta\alpha \int_{-1}^{+1} -Pr^{-1}\left(\frac{\partial\psi_1}{\partial\sigma}(1, \sigma)\right)^2 \sigma \, d\sigma.$$

F_3 is the result of the friction forces acting on the drop. The corresponding fractions of the total drag coefficient C_D on the drop is calculated from $C_D = C_{DP} + C_{DM} + C_{DF}$. C_{DP} corresponds to the pressure forces, C_{DM} corresponds to the mass transfer forces and C_{DF} corresponds to the friction forces. Using the asymptotic expansion [11] the following results is established where C_{DS} is the Stokes drag coefficient corresponding to an isothermal flow without mass transfer:

$$\frac{C_{DP}}{C_{DS}} = \frac{1}{9}\left[2H_2(B, Pr) - H_1(B, Pr)\left[\frac{d^3Z}{dr_1^3}(1) + 4\frac{dZ(1)}{dr_1}\right) - 4 + Pr^{-1}\left(2H_2(B, Pr) + 2 + H_1(B, Pr)\frac{d^2Z}{dr_1^2}(1)\log(1+B)\right)\right] + \mathrm{Ord}(Pe)$$

$$\frac{C_{DH}}{C_{DS}} = -\frac{4}{9}Pr^{-1}[H_2(B, Pr) + 2 + H_1(B, Pr)Z(1)\log(1+B)] + \mathrm{Ord}(Pe)$$

$$\frac{C_{DF}}{C_{DS}} = \frac{1}{9}H_1(B, Pr)\left[2\frac{d^2Z(1)}{dr_1^2} - 4Z(1)\right] + \mathrm{Ord}(Pe).$$

The functions $H_1(B, Pr)$ and $H_2(B, Pr)$ were computed and the variations of the four functions C_{DP}/C_{DS}, C_{DM}/C_{DS}, C_{DF}/C_{PS} and C_D/C_{DS} as functions of the superheat parameter B are presented for different Prandtl numbers on figures 4–7. When $B = 0$ the Stokes result is obtained: $C_D/C_{DS} = 1$. The evaporation of a drop results in a decrease of its drag coefficient from its Stokes value. The pressure term remains practically constant and the friction term decreases when the

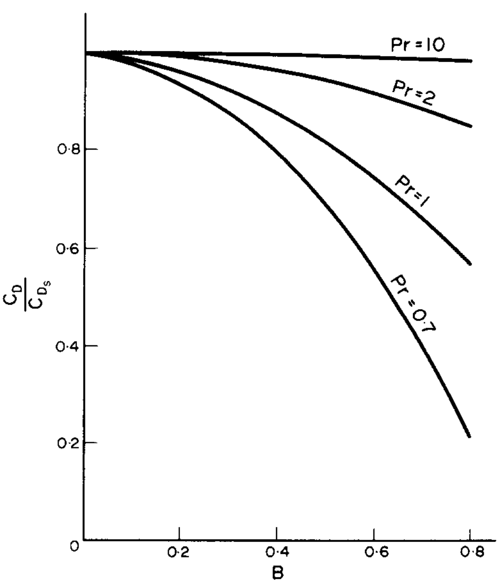

Figure 4. Total drag coefficient as a function of the superheat parameter B and the Prandtl number Pr.

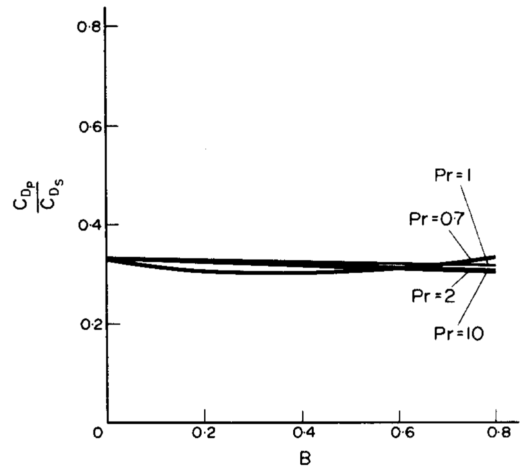

Figure 5. Pressure term of the drag coefficient as a function of the superheat parameter B and the Prandtl number Pr.

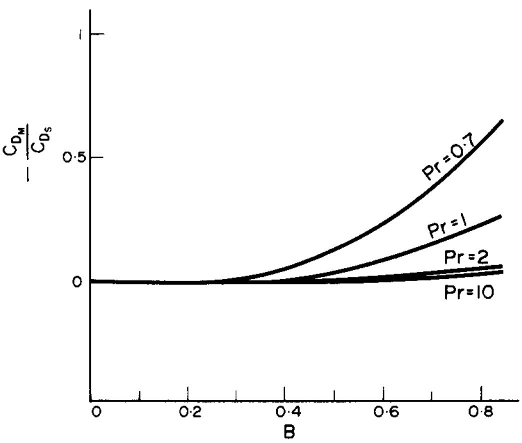

Figure 6. Mass transfer term of the drag coefficient as a function of the superheat parameter B and the Prandtl number Pr.

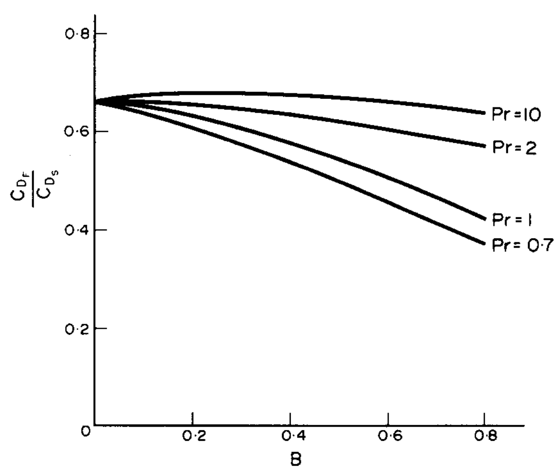

Figure 7. Friction term of the drag coefficient as a function of the superheat parameter B and the Prandtl number Pr.

superheat increases. Resistance due to mass transfer increases a little ($C_{DM} > 0$) when this mass transfer is low; when it becomes more important this resistance decreases ($C_{DM} < 0$).

CONCLUSION

The evaporation of a water drop in a superheated steam flow is a pure multiphase heat and mass transfer problem which can be handled by singular perturbation methods. Three regions must be considered in order to get a correct approximation of the complete solution in the form of asymptotic expansions using the Peclet number of the drop as a small parameter.

The determination of the first terms of the expansions of the temperature and of the Stokes stream function leads to semianalytical expressions for the Nusselt number and the drag coefficient of the drop. From these expressions it can be concluded that the Nusselt number as well as the drag coefficient are decreasing functions of the superheat of the steam.

Such results, which were confirmed by the experimental results (Montluçon 1972) give information on the interactions between phases, which can be used in the theoretical modeling of two-phase flows.

Acknowledgements—The author wishes to express his special gratitude to Professor A. Craya, Université Scientifique et Médicale de Grenoble. His constructive comments, criticism and suggestions have contributed to the completion of this study.

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Résumé—On étudie par une méthode de perturbation singulière les transferts de chaleur et de masse autour d'une goutte d'eau s'évaporant dans un écoulement de vapeur surchauffée.

Le schéma de perturbation comporte trois domaines. On calcule le nombre de Nüsselt et le coefficient de traînée.

Auszug—Der Wärme- und Stoffaustausch um einen Wassertropfen, der im überhitzten Dampfstrom verdampft, wird mit Hilfe singulärer Perturbationsmethoden untersucht.

Das Perturbationsschema umfasst drei Gebiete. Mehrere Ergebnisse, die die Nusseltsche Kennzahl und den Widerstandsköffizienten des Tropfens betreffen, werden mitgeteilt.

Резюме—Методом единичных возмущений изучена проблема тепломассопереноса вблизи испаряющейся капли, воды, находящейся в потоке перегретого пара.

Схема таких возмущений включала в себя три некоторых участка. В работе представлены частичные результаты относительно критерия Нуссельта и коэффициента уноса капель.